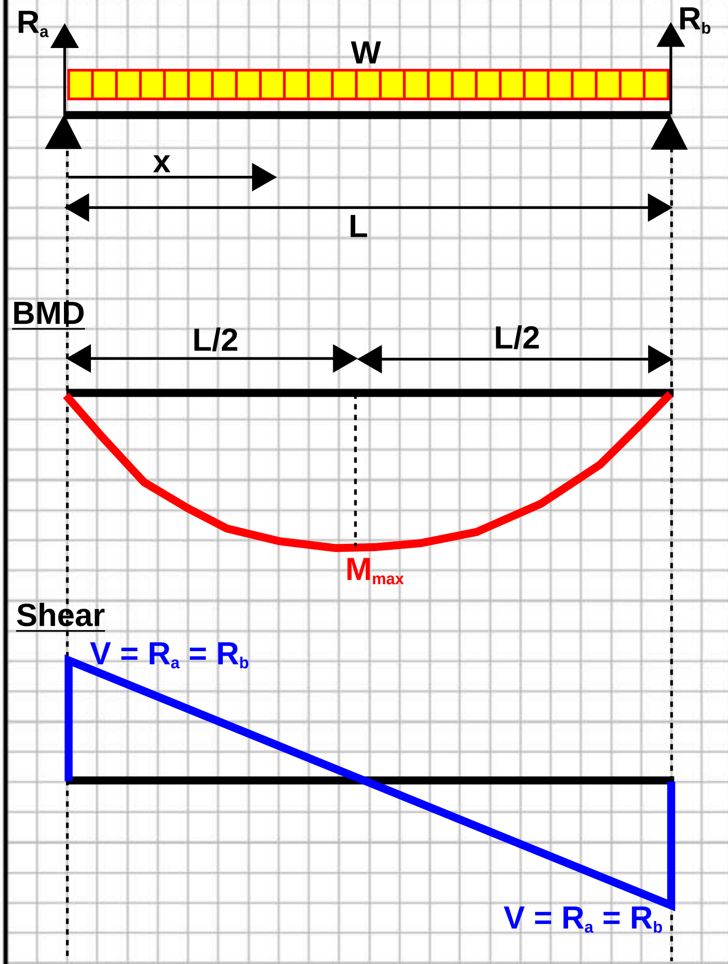
	Design Spreadsheet:	Date:
	Beam Analysis v1.0.xlsm	Oct 2021
		By: AL
	Subtitle:	Version:
	Beam Analysis: Bending Moments, Shear & Deflections	1

Simply Supported Beams

Uniformly Distributed Load Along Entire Beam



Inputs:

W : uniformly distributed load (kN/m)

L : Length of beam (m)

E : Modulus of Elasticity (N/mm^2)

I : Second Moment of Area (mm^4)

(for E & I you'll need to convert to kN and m)

Reactions:

$$R_a = R_b = \frac{WL}{2}$$

Moments:

$$M = \frac{Wx}{2}(L - x)$$

$$M_{max} = \frac{WL^2}{8} \text{ at midspan}$$

Shear:

$$V = W(\frac{L}{2} - x)$$

$$V_{max} = \frac{WL}{2}$$

Deflection:

$$\delta = \frac{Wx}{24EI}(L^3 - 2Lx^2 + x^3)$$

$$\delta_{max} = \frac{5WL^4}{384EI} \text{ at midspan}$$

Design Spreadsheet:	Date:
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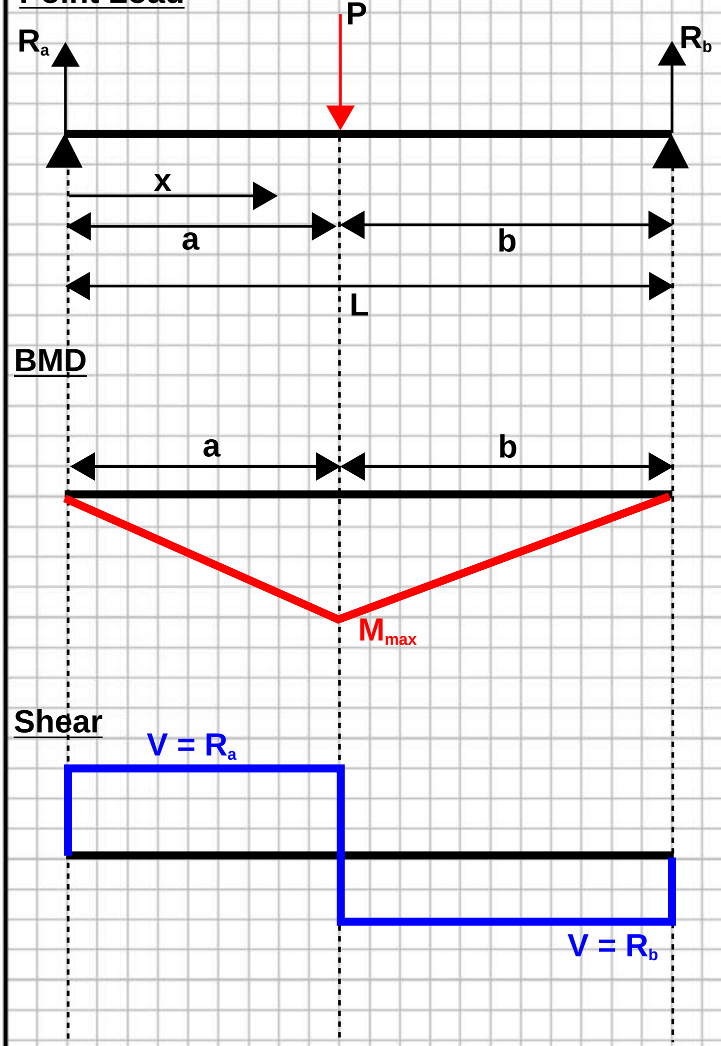
Beam Analysis: Bending Moments, Shear & Deflections	Version: 1

Simply Supported Beams

Point Load



Inputs:

P : Point Load (kN)
L : Length of beam (m)
a & b: Distances to the point load (m)
E : Modulus of Elasticity (N/mm^2)
I : Second Moment of Area (mm^4)
(for E & I you'll need to convert to kN and m)

Reactions:

$$R_a = \frac{Pb}{L} \qquad R_b = \frac{Pa}{L}$$

Moments:

$$M = \frac{Pbx}{L} \quad for : x \le a$$

$$M = \frac{Pa(L-x)}{L} \quad for : x > a$$

Shear:

$$V = R_a \quad for : x \le a$$

$$V = -R_b \quad for : x > a$$

Deflection:

$$\delta = \frac{Pbx}{6EIL}(L^2 - b^2 - x^2) \quad for : x \le a$$

$$\delta = \frac{Pa(L-x)}{6EIL}(2Lx - x^2 - a^2) \quad for : x > a$$

$$\delta_{max} = \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EIL}$$

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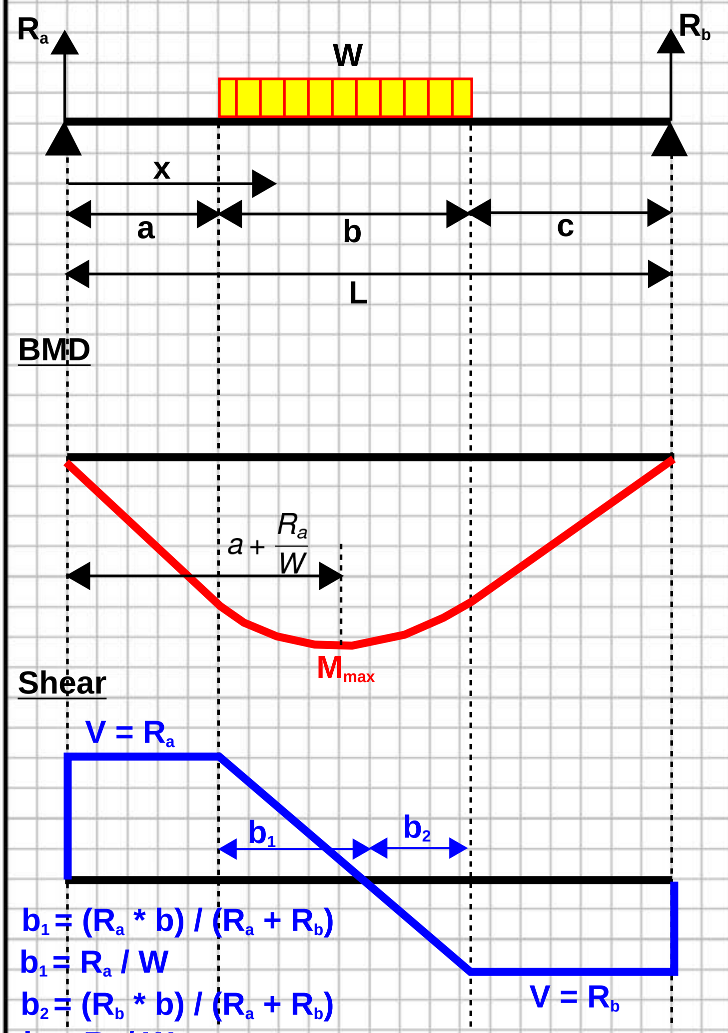
Simply Supported Beams

Partial UDL



$b_1 = (R_a * b) / (R_a + R_b)$
$b_1 = R_a / W$
$b_2 = (R_b * b) / (R_a + R_b)$
$b_2 = R_b / W$

Inputs:
W: Uniformly distributed load (kN/m)
L : Length of beam (m)
a,b & c: Distances and length of the UDL
E : Modulus of Elasticity (N/mm^2)
I : Second Moment of Area (mm^4)
(for E & I you'll need to convert to kN and m)

Reactions:

$$R_a = \frac{Wb}{2L}(2c+b)$$

$$R_b = \frac{Wb}{2L}(2a+b)$$

Moments:

$$M_{max} = R_a\left(a+\frac{R_a}{2W}\right)$$

$$M = R_a x \quad for: x \leq a$$

$$M = R_a x - \frac{W}{2}(x-a)^2 \quad for: a < x \leq a+b$$

$$M = R_b(L-x) \quad for: x > a+b$$

Shear:

$$V = R_a \quad for: x \leq a$$

$$V = R_a - W(x-a) \quad for: a < x \leq a+b$$

$$V = R_b \quad for: x > a+b$$

Deflection:

$$\gamma = \frac{R_a(a+b)^3}{6} - \frac{Wb^4}{24} - \frac{R_b}{6}(L-a-b)^3$$

$$\phi = \frac{R_a(a+b)^2}{2} - \frac{Wb^3}{6} + \frac{R_b}{2}(L-a-b)^2$$

$$C_1 = \frac{\gamma - \phi([a+b]-L)}{([a+b]-L)-(a+b)}$$

$$E_1 = C_1 + \phi$$

please do not confuse E_1 with the modulus of elasticity E

$$\delta = \frac{1}{EI}\left(\frac{R_a x^3}{6} + C_1 x\right) \quad for: x \leq a$$

$$\delta = \frac{1}{EI}\left(\frac{R_a x^3}{6} - \frac{W}{24}(x-a)^4 + C_1 x\right) \quad for: a < x \leq a+b$$

$$\delta = \frac{1}{EI}\left(\frac{R_b}{6}(L-x)^3 + E_1 x - E_1 L\right) \quad for: x > a+b$$

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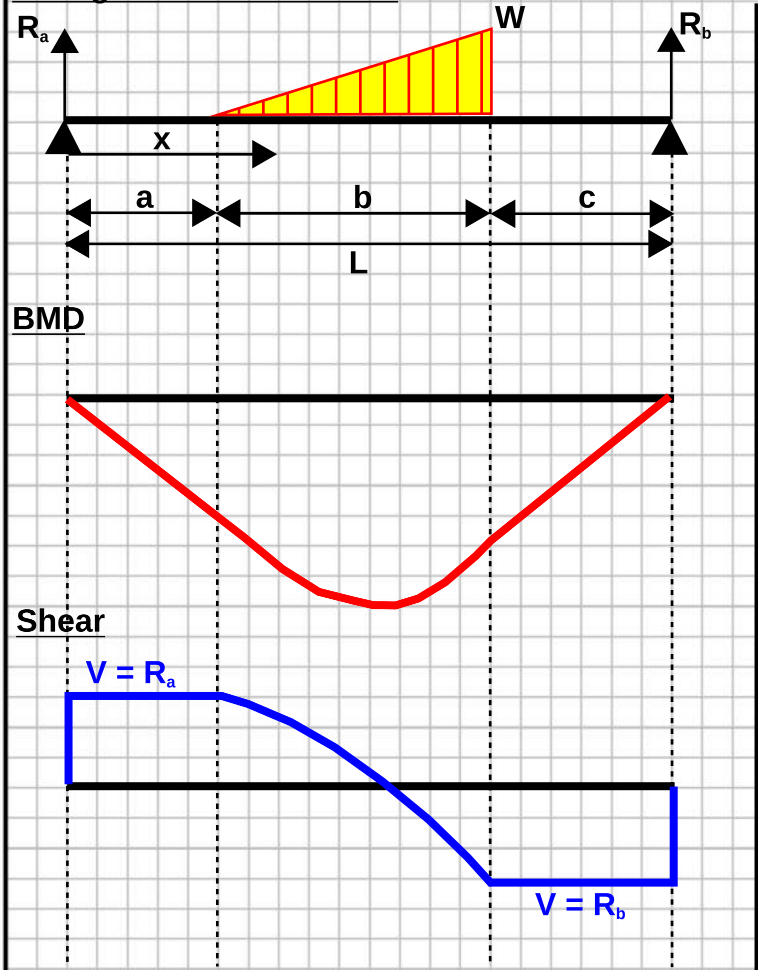
Simply Supported Beams

Triangular Load on Beam



Inputs:

W : uniformly distributed load (kN/m)
L : Length of beam (m)
E : Modulus of Elasticity (N/mm²)
I : Second Moment of Area (mm⁴)
(for E & I you'll need to convert to kN and m)

Reactions:

$$R_a = \frac{Wb}{2} - R_b$$

$$R_b = \frac{Wb}{2L}(a + \frac{2}{3}b)$$

Moments:

$$M = R_a x \quad for : x \le a$$

$$M = R_a x - \frac{(x-a)^3}{6b} \quad for : a < x \le a + b$$

$$M = R_a x - \frac{Wb}{2}(x - a - \frac{2}{3}b) \quad for : x > a + b$$

Shear:

$$V = R_a \quad for : x \le a$$

$$V = R_a - \frac{W(x-a)^2}{2b} \quad for : a < x \le a + b$$

$$V = -R_b \quad for : x > a + b$$

Deflection:

$$\lambda = \frac{Wb^4}{120} - \frac{Wb}{2}(\frac{(a+b)^3}{6} - \frac{a(a+b)^2}{2} - \frac{b(a+b)^2}{3})$$

$$\phi = \frac{Wb^3}{24} - \frac{Wb}{2}(\frac{(a+b)^2}{2} - a(a+b) - \frac{2}{3}b(a+b))$$

$$\gamma = \frac{Wb}{2}(\frac{L^3}{6} - \frac{aL^2}{2} - \frac{bL^2}{3}) - \frac{R_a L^3}{6}$$

$$E_1 = \frac{\lambda + \gamma - \phi(a+b)}{L}$$

$$C_1 = \phi + E_1$$

$$E_2 = \gamma - E_1 L$$

please do not confuse E_1 & E_2 with the modulus of elasticity E

$$\delta = \frac{1}{EI}(\frac{R_a x^3}{6} + C_1 x) \quad for : x \le a$$

$$\delta = \frac{1}{EI}(\frac{R_a x^3}{6} - \frac{W(x-a)^5}{120b} + C_1 x) \quad for : a < x \le a + b$$

$$\delta = \frac{1}{EI}(\frac{R_a x^3}{6} - \frac{Wb}{2}(\frac{x^3}{6} - \frac{ax^2}{2} - \frac{bx^2}{3}) + E_1 x + E_2) \quad for : x > a + b$$

If you want to model a triangular load going in the other direction (i.e. starts at W and drops to zero when going left-right). Simply set the following variables before running through the equations above

x = L - x
c = a
a = L - a - b

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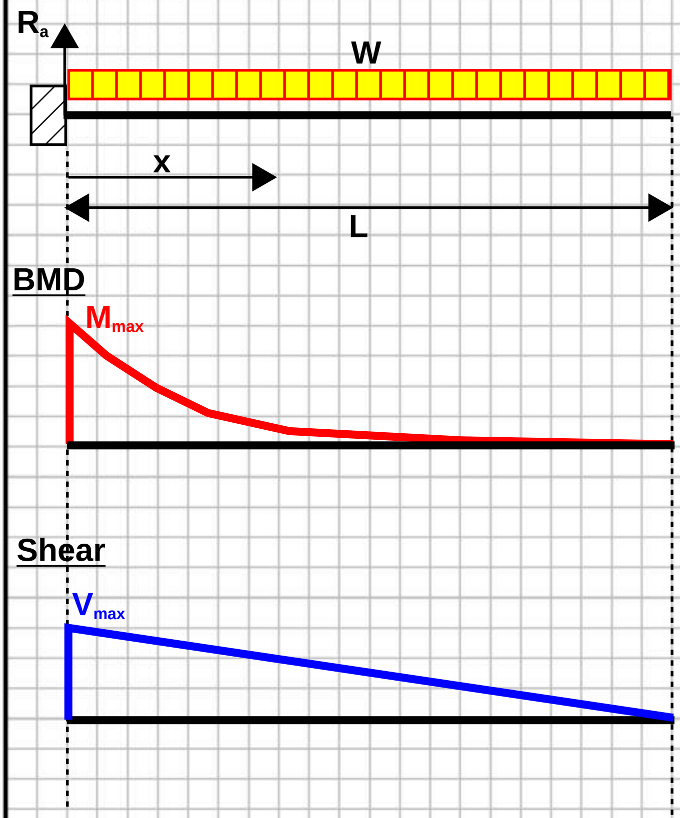
Cantilever Beams

Uniformly Distributed Load Along Entire Beam



Inputs:

W : uniformly distributed load (kN/m)

L : Length of beam (m)

E : Modulus of Elasticity (N/mm^2)

I : Second Moment of Area (mm^4)

(for E & I you'll need to convert to kN and m)

Reactions:

$$R_a = WL$$

Moments:

$$M = \frac{W(L-x)^2}{2} \qquad M_{max} = \frac{WL^2}{2} \text{ at support}$$

Shear:

$$V = W(L-x) \qquad V_{max} = WL$$

Deflection:

$$\delta = \frac{W}{24EI}((L-x)^4 - 4L^3(L-x) + 3L^4)$$

$$\delta_{max} = \frac{WL^4}{8EI} \text{ at end of cantilever}$$

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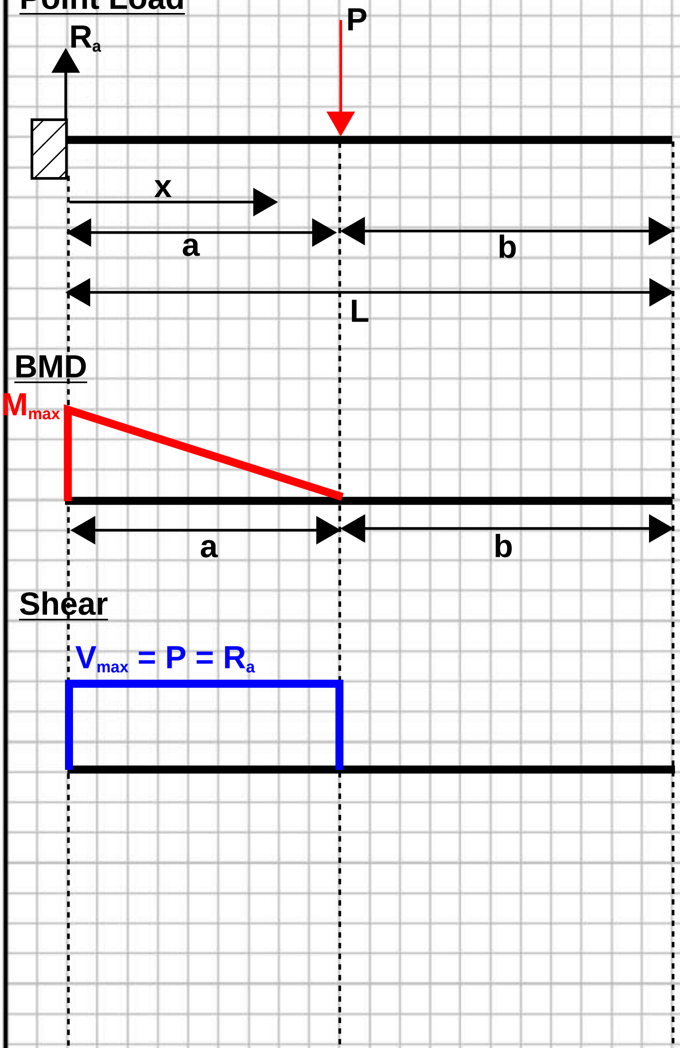
Cantilever Beams

Point Load



Inputs:
P : Point Load (kN)
L : Length of beam (m)
a & b: Distances to the point load (m)
E : Modulus of Elasticity (N/mm^2)
I : Second Moment of Area (mm^4)
(for E & I you'll need to convert to kN and m)

Reactions:

$$R_a = P$$

Moments:

$$M = P(a - x)$$

$$M = 0 \quad for : x > a$$

$$M_{max} = Pa \text{ at support}$$

Shear:

$$V = P \quad for : x \leq a \qquad V = 0 \quad for : x > a$$

Deflection:

$$\delta = \frac{Px^2}{6EI}(3a - x) \quad for : x \leq a$$

$$\delta = \frac{Pa^2}{6EI}(3L - 3(L - x) - a) \quad for : x > a$$

$$\delta_{max} = \frac{Pa^2}{6EI}(3L - a) \text{ at end of cantilever}$$

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		By: AL
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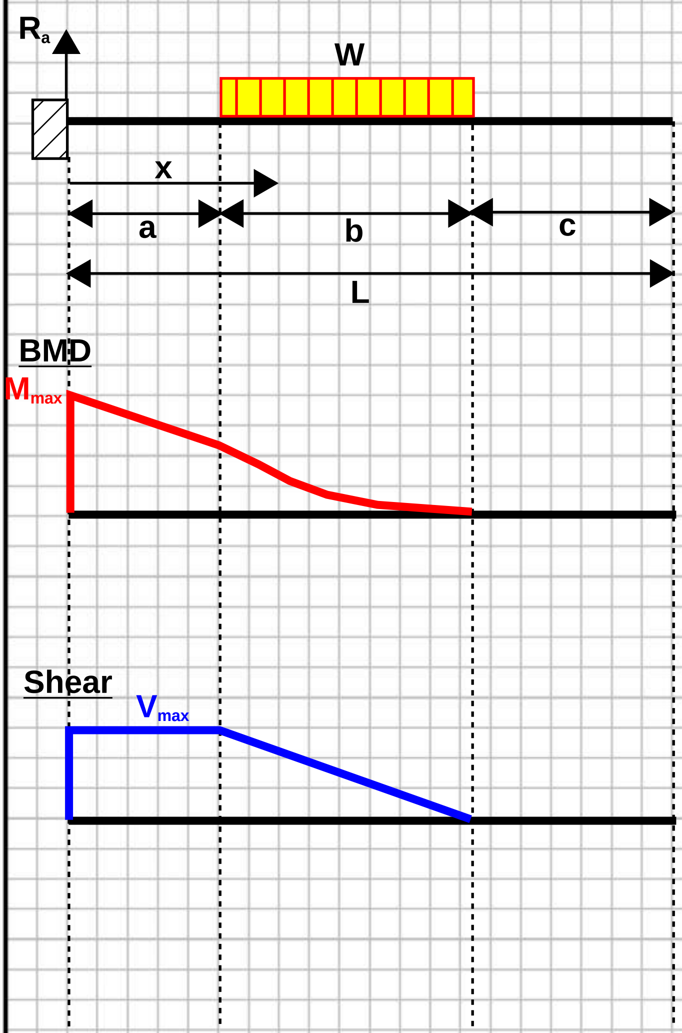
Cantilever Beams

Partial UDL



Inputs:
W: Uniformly distributed load (kN/m)
L : Length of beam (m)
a,b & c: Distances and length of the UDL
E : Modulus of Elasticity (N/mm^2)
I : Second Moment of Area (mm^4)
(for E & I you'll need to convert to kN and m)

Reactions:

$$R_a = Wb$$

Moments:

$$M = Wb(a - x + \frac{b}{2}) \quad for: x \leq a$$

$$M = \frac{W}{2}(x - a - b)^2 \quad for: a < x \leq a + b$$

$$M = 0 \quad for: x \geq a + b$$

Shear:

$$V = Wb \quad for: x \leq a$$

$$V = W(b - (x - a)) \quad for: a < x \leq a + b$$

$$V = 0 \quad for: x \geq a + b$$

Deflection:

$$C_1 = \frac{Wb}{2}(a + \frac{b}{2})^2 \quad C_2 = \frac{-Wb}{6}(a + \frac{b}{2})^3$$

$$\delta = \frac{1}{EI}[\frac{Wb}{6}(a - x + \frac{b}{2})^3 + C_1 x + C_2] \quad for: x \leq a$$

$$D_1 = \frac{Wb^3}{24} + \frac{Wb}{2}(a + \frac{b}{2})^2$$

$$D_2 = \frac{-Wb^4}{48} - \frac{Wb}{6}(a + \frac{b}{2})^3 - \frac{Wb^3 a}{24}$$

$$\delta = \frac{1}{EI}[\frac{W}{24}(x - a - b)^4 + D_1 x + D_2] \quad for: a < x \leq a + b$$

$$\delta = \frac{1}{EI}[D_1 x + D_2] \quad for: x > a + b$$

$$\delta_{max} = \frac{1}{EI}[D_1 L + D_2] \quad \text{at end of cantilever}$$

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	Subtitle:	By: AL
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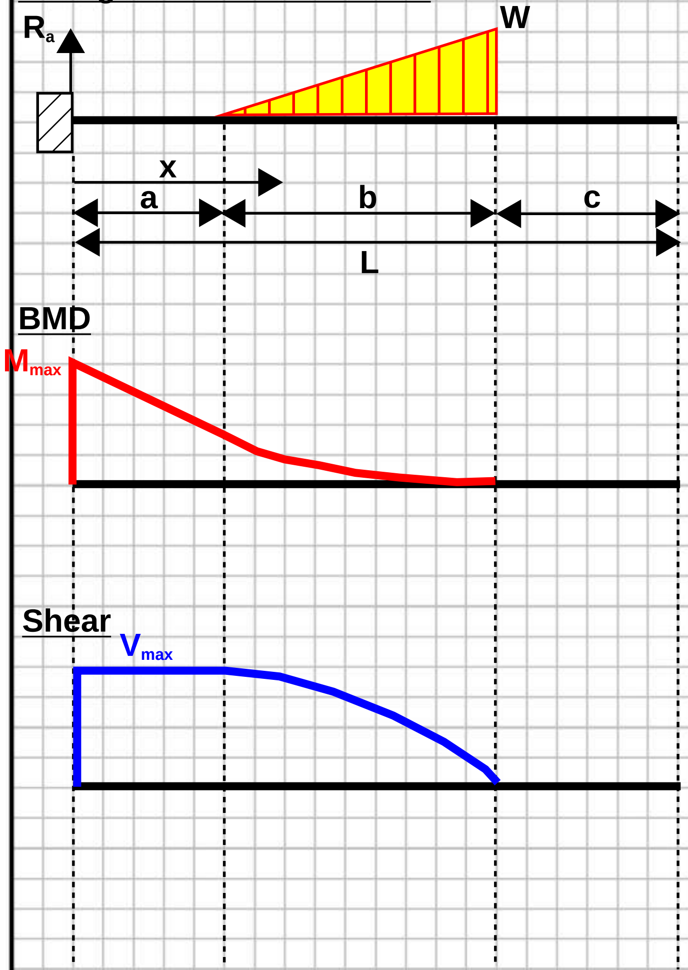
Cantilever Beams

Triangular Load on Beam 1



Inputs:

W : uniformly distributed load (kN/m)
L : Length of beam (m)
E : Modulus of Elasticity (N/mm^2)
I : Second Moment of Area (mm^4)
(for E & I you'll need to convert to kN and m)

Reactions:

$$R_a = \frac{Wb}{2}$$

Moments:

$$M = R_a(a - x + \frac{2}{3}b) \quad for: x \leq a$$

$$M = R_a(\frac{2}{3}b + a - x) + \frac{W(x-a)^3}{6b} \quad for: a < x \leq a+b$$

$$M = 0 \quad for: x > a+b$$

Shear:

$$V = R_a \quad for: x \leq a$$

$$V = R_a - \frac{W(x-a)^2}{2b} \quad for: a < x \leq a+b$$

$$V = 0 \quad for: x > a+b$$

Deflection:

$$E_1 = R_a(\frac{2b(a+b)}{3} + a(a+b) - \frac{(a+b)^2}{2}) + \frac{Wb^3}{24}$$

$$E_2 = -E_1(a+b)$$

$$\delta = \frac{1}{EI}(R_a(\frac{ax^2}{2} - \frac{x^3}{6} + \frac{bx^2}{3})) \quad for: x \leq a$$

$$\delta = \frac{1}{EI}(R_a(\frac{bx^2}{3} + \frac{ax^2}{2} - \frac{x^3}{6}) + \frac{W(x-a)^5}{120b})$$

$$for: a < x \leq a+b$$

$$\delta = \frac{1}{EI}(E_1 x + E_2 + R_a(\frac{b(a+b)^2}{3} + \frac{a(a+b)^2}{2} - \frac{(a+b)^3}{6}) + \frac{Wb^4}{120})$$

$$for: x > a+b$$

Design Spreadsheet: Beam Analysis v1.0.xlsm

Subtitle: Beam Analysis: Bending Moments, Shear & Deflections

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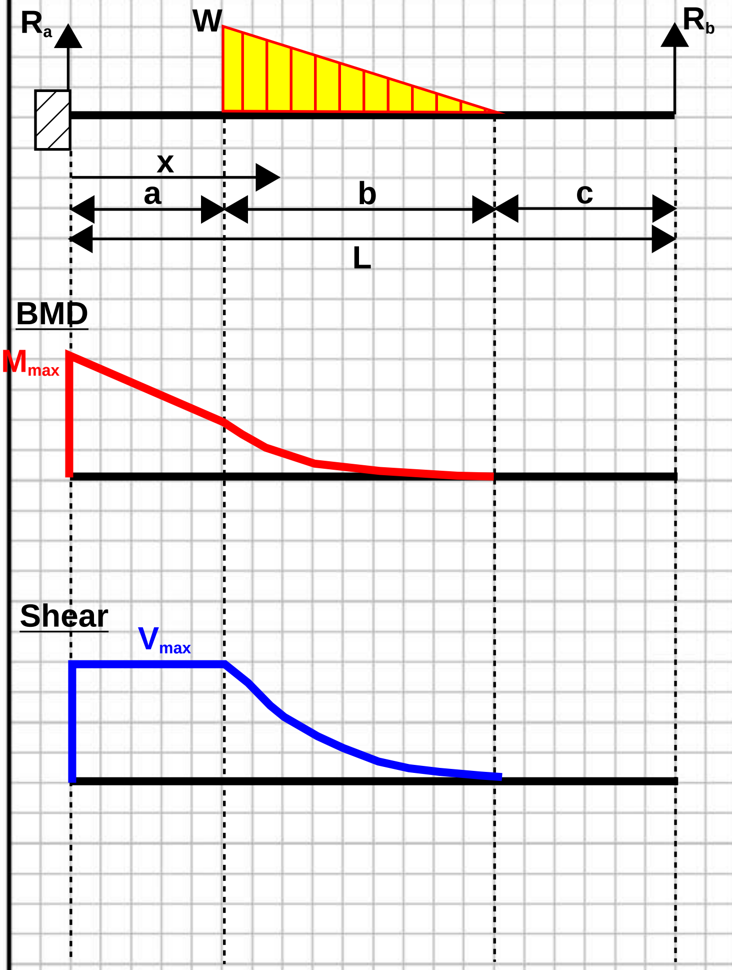
By: AL

Version: 1

Simply Supported Beams

Triangular Load on Beam 2



Inputs:

W : uniformly distributed load (kN/m)

L : Length of beam (m)

E : Modulus of Elasticity (N/mm²)

I : Second Moment of Area (mm⁴)

(for E & I you'll need to convert to kN and m)

Reactions:

$$R_a = \frac{Wb}{2}$$

Moments:

$$M = R_a\left(a - x + \frac{b}{3}\right) \quad for : x \leq a$$

$$M = \frac{W(b + a - x)^3}{6b} \quad for : a < x \leq a + b$$

$$M = 0 \quad for : x > a + b$$

Shear:

$$V = R_a \quad for : x \leq a$$

$$V = R_a - W(x - a) + \frac{W(x - a)^2}{2b} \quad for : a < x \leq a + b$$

$$V = 0 \quad for : x > a + b$$

Deflection:

$$E_1 = R_a\left(\frac{ba}{3} + \frac{a^2}{2}\right) + \frac{Wb^3}{24}$$

$$E_2 = R_a\left(\frac{a^3}{3} + \frac{ba^2}{6}\right) - \frac{Wb^4}{120} - E_1 a$$

$$\delta = \frac{1}{EI}\left(R_a\left(\frac{ax^2}{2} - \frac{x^3}{6} + \frac{bx^2}{6}\right)\right) \quad for : x \leq a$$

$$\delta = \frac{1}{EI}\left(\frac{W}{120b}(b + a - x)^5 + E_1 x + E_2\right) \quad for : a < x \leq a + b$$

$$\delta = \frac{1}{EI}(E_1 x + E_2) \quad for : x > a + b$$

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		By: AL
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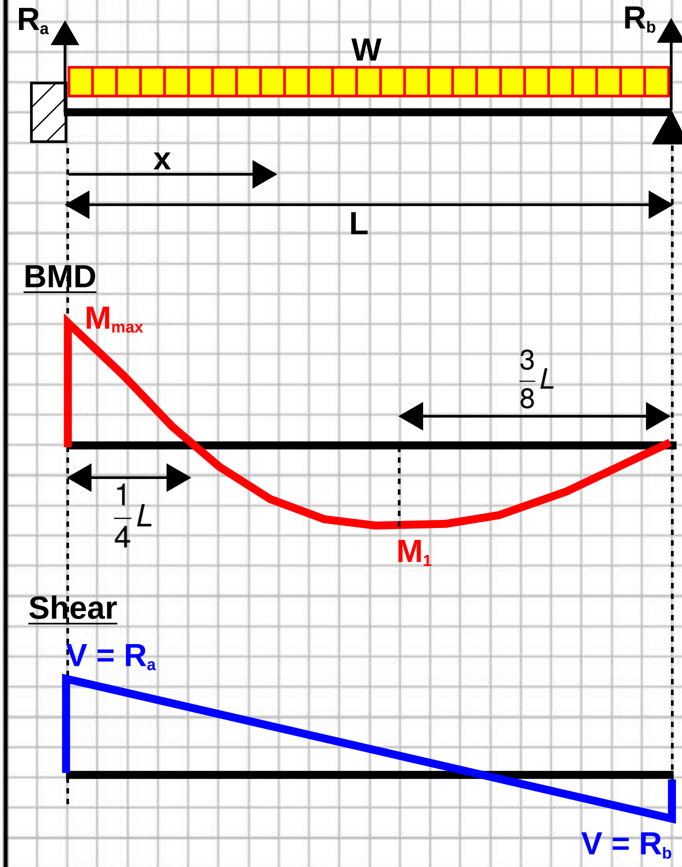
Propped Cantilever Beams

Uniformly Distributed Load Along Entire Beam



Inputs:

W : uniformly distributed load (kN/m)

L : Length of beam (m)

E : Modulus of Elasticity (N/mm^2)

I : Second Moment of Area (mm^4)

(for E & I you'll need to convert to kN and m)

Reactions:

$$R_a = \frac{5WL}{8}$$

$$R_b = \frac{3WL}{8}$$

Moments:

$$M = R_b(L-x) - \frac{W(L-x)^2}{2}$$

$$M_{max} = \frac{WL^2}{8} \qquad M_1 = \frac{9}{128}WL^2$$

Shear:

$$V = R_b - W(L-x)$$

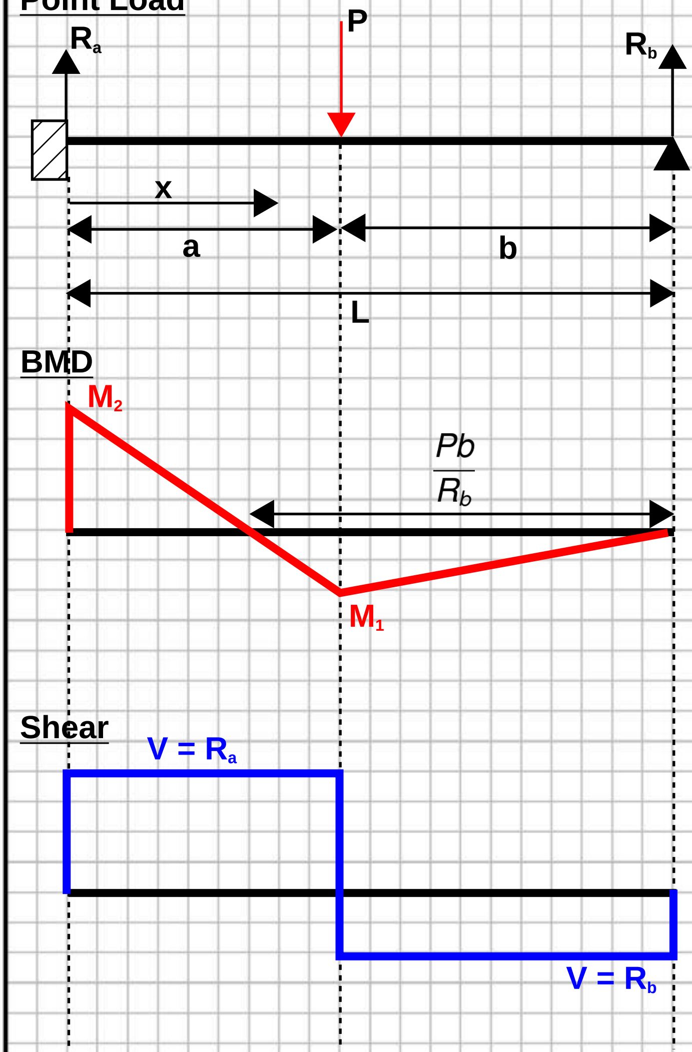
Deflection:

$$\delta = \frac{W(L-x)}{48EI}(L^3 - 3L(L-x)^2 + 2(L-x)^3)$$

$$\delta_{max} = \frac{WL^4}{185EI}$$

Propped Cantilever Beams

Point Load



Inputs:

P : Point Load (kN)
L : Length of beam (m)
a & b: Distances to the point load (m)
E : Modulus of Elasticity (N/mm^2)
I : Second Moment of Area (mm^4)
(for E & I you'll need to convert to kN and m)

Reactions:

$$R_a = \frac{Pb}{2L^3}(3L^2 - b^2)$$

$$R_b = \frac{Pa^2}{2L^3}(b + 2L)$$

Moment:

$$M_1 = R_b b$$

$$M_2 = \frac{Pab}{2L^2}(b + L)$$

$$M = R_b(L - x) - P((L - x) - b) \quad for : x \leq a$$

$$M = R_b(L - x) \quad for : x > a$$

Shear:

$$V = R_a \quad for : x \leq a$$

$$V = R_b \quad for : x > a$$

Deflection:

$$\delta = \frac{Pbx^2}{12EIL^3}(3L^2(L - x) - b^2(L - x) - 2b^2L) \quad for : x \leq a$$

$$\delta = \frac{Pa^2(L - x)}{12EIL^3}(3bL^2 - 2L(L - x)^2 - b(L - x)^2) \quad for : x > a$$

$$\delta_{max} = \frac{Pb^2a^3}{12EIL^3}(3L + b)$$

Design Spreadsheet:

Beam Analysis v1.0.xlsm

Subtitle:

Beam Analysis: Bending Moments, Shear & Deflections

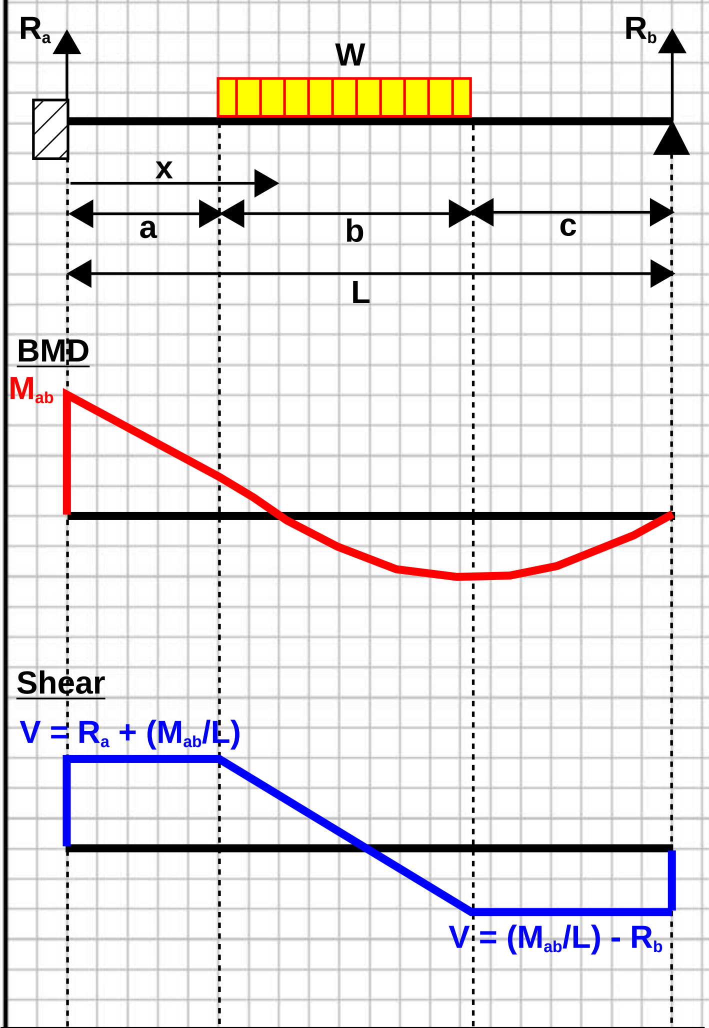
Date: **Oct 2021**

By: **AL**

Version: **1**

Propped Cantilever Beams

Partial UDL



Inputs:

W: Uniformly distributed load (kN/m)

L : Length of beam (m)

a,b & c: Distances and length of the UDL

E : Modulus of Elasticity (N/mm^2)

I : Second Moment of Area (mm^4)

(for E & I you'll need to convert to kN and m)

Intermediate Reactions:

$$R_a = \frac{Wb}{2L}(2c+b) \qquad R_b = \frac{Wb}{2L}(2a+b)$$

final reactions found from the shear force diagram at x = 0 and x = L

Moments:

$$d = b + c$$

$$M_{ab} = \left| \frac{-W}{8L^2}(d^2 - c^2)(2L^2 - c^2 - d^2) \right|$$

$$M' = \frac{M_{ab}(L-x)}{L}$$

$$M = M' - R_a x \quad for: x \le a$$

$$M = M' - (R_a x - \frac{W}{2}(x-a)^2) \quad for: a < x \le a+b$$

$$M = M' - R_b(L-x) \quad for: x \ge a+b$$

Shear:

$$V = R_a + \frac{M_{ab}}{L} \quad for: x \le a$$

$$V = [R_a + \frac{M_{ab}}{L}] - |R_a + R_b| \frac{(x-a)}{b} \quad for: a < x \le a+b$$

$$V = -(R_b - \frac{M_{ab}}{L}) \quad for: x > a+b$$

Deflection:

$$C_1 = \frac{M_{ab}L}{2} \qquad C_2 = \frac{-M_{ab}L^2}{6}$$

$$E_1 = \frac{-R_b(L-a-b)^2}{2} - \frac{R_a(a+b)^2}{2} + \frac{wb^3}{6} + C_1$$

$$E_2 = -E_1 L$$

$$\delta = \frac{1}{EI}\left(\frac{M_{ab}(L-x)^3}{6L} - \frac{R_a x^3}{6} + C_1 x + C_2\right)$$

$for: x \le a$

$$\delta = \frac{1}{EI}\left(\frac{M_{ab}(L-x)^3}{6L} - \frac{R_a x^3}{6} + \frac{W(x-a)^4}{24} + C_1 x + C_2\right) \quad for: a < x \le a+b$$

$$\delta = \frac{1}{EI}\left(\frac{M_{ab}(L-x)^3}{6L} - \frac{R_b(L-x)^3}{6} + E_1 x + E_2\right) \quad for: x > a+b$$

please do not confuse E_1 & E_2 with the modulus of elasticity E

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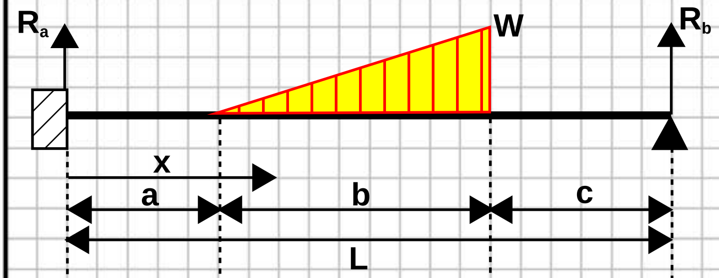
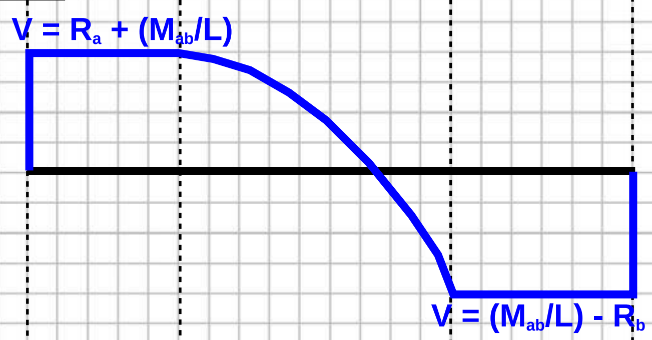
Simply Supported Beams

Triangular Load on Beam 1



BMD

Shear



Inputs:

W : uniformly distributed load (kN/m)

L : Length of beam (m)

E : Modulus of Elasticity (N/mm^2)

I : Second Moment of Area (mm^4)

(for E & I you'll need to convert to kN and m)

Intermediate Reactions:

$$R_b = \frac{Wb}{2L}\left(a + \frac{2}{3}b\right) \qquad R_a = \frac{Wb}{2} - R_b$$

final reactions found from the shear force diagram at x = 0 and x = L

Moments:

$$M_{ab1} = \frac{Wb}{120L^2}$$

$$M_{ab2} = 30a^3 + 30a^2(2b - 3L) + 15a(3b^2 - 8bL + 4L^2)$$

$$M_{ab3} = b(12b^2 - 45bL + 40L^2)$$

$$M_{ab} = M_{ab1} * (M_{ab2} + M_{ab3})$$

$$M' = \frac{M_{ab}(L - x)}{L}$$

$$M = M' - R_a x \quad for : x \le a$$

$$M = M' - (R_a x) + \frac{W}{6b}(x - a)^3 \quad for : a < x \le a + b$$

$$M = M' - R_b(L - x) \quad for : x > a + b$$

Shear:

$$V = R_a + \frac{M_{ab}}{L} \quad for : x \le a$$

$$V = R_a + \frac{M_{ab} - M_{ba}}{L} - \frac{W}{2b}(x - a)^2 \quad for : a < x \le a + b$$

$$V = -\left(R_b - \frac{M_{ab}}{L}\right) \quad for : x > a + b$$

Deflection:

$$\delta = \frac{M_{ab}(L - x)^3}{6L} - \frac{R_a x^3}{6} + \frac{M_{ab} L x}{2} - \frac{M_{ab} L^2}{6} \quad for : x \le a$$

$$\delta = \frac{M_{ab}(L - x)^3}{6L} - \frac{R_a x^3}{6} + \frac{W(x - a)^5}{120b} + \frac{M_{ab} L x}{2} - \frac{M_{ab} L^2}{6} \quad for : a < x \le a + b$$

$$\delta = \frac{M_{ab}(L - x)^3}{6L} - \frac{R_b(L - x)^3}{6} + E_1 x + E_2 \quad for : x > a + b$$

$$E_1 = \frac{Wb^3}{24} + \frac{M_{ab} L}{2} - \frac{R_a(a + b)^2}{2} - \frac{R_b(L - a - b)^2}{2}$$

$$E_2 = -E_1 L$$

please do not confuse E_1 & E_2 with the modulus of elasticity E

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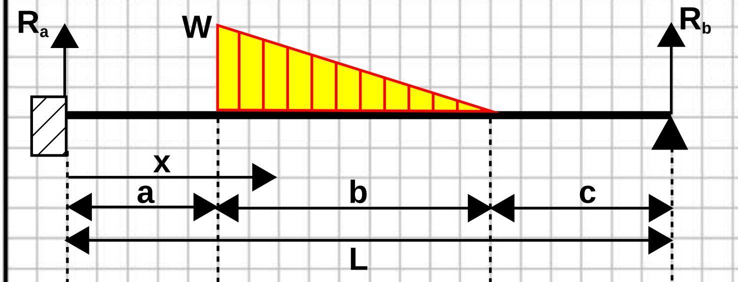
Simply Supported Beams

Triangular Load on Beam 2



BMD

Shear

Inputs:

W : uniformly distributed load (kN/m)
L : Length of beam (m)
E : Modulus of Elasticity (N/mm^2)
I : Second Moment of Area (mm^4)
(for E & I you'll need to convert to kN and m)

Intermediate Reactions:

$$R_b = \frac{Wb}{2L}(a + \frac{b}{3}) \qquad R_a = \frac{Wb}{2} - R_b$$

final reactions found from the shear force diagram at x = 0 and x = L

Moments:

$$M_{ab1} = \frac{Wb}{120L^2}$$

$$M_{ab2} = 30a^2 + 30a^2(b - 3L) + 15a(b - 2L)^2$$

$$M_{ab3} = b(3b^2 - 15bL + 20L^2)$$

$$M_{ab} = M_{ab1} * (M_{ab2} + M_{ab3})$$

$$M' = \frac{M_{ab}(L - x)}{L}$$

$$M = M' - R_a x \quad for: x \leq a$$

$$M = M' - R_a x + (x - a)^2(\frac{W(a + b - x)}{2b} + \frac{W}{3} - \frac{W(a + b - x)}{3b}) \quad for: a < x \leq a + b$$

$$M = M' - R_b(L - x) \quad for: x > a + b$$

Shear:

$$V = R_a + \frac{M_{ab}}{L} \quad for: x \leq a$$

$$V = R_a + \frac{M_{ab} - M_{ba}}{L} - \frac{(x - a)}{2}(W + \frac{W(a + b - x)}{b}) \quad for: a < x \leq a + b$$

$$V = -(R_b - \frac{M_{ab}}{L}) \quad for: x > a + b$$

Deflection:

$$D_1 = \frac{M_{ab}L}{2} - \frac{Wa^4}{24b} - \frac{Wa^3}{6}$$

$$D_2 = \frac{M_{ab}La}{2} - \frac{M_{ab}L^2}{6} + \frac{Wa^4}{b}(-\frac{a}{24} - \frac{b}{8}) + \frac{Wa^5}{120b} - D_1 a$$

$$E_1 = -\frac{R_b(L - a - b)^2}{2} - \frac{R_a(a + b)^2}{2} + \frac{W(a^4 + 4a^3b + 3b^4)}{24b} + D_1$$

$$E_2 = -E_1 L$$

$$\delta = \frac{1}{EI}(\frac{M_{ab}(L - x)^3}{6L} - \frac{R_a x^3}{6} + \frac{M_{ab}L}{2} - \frac{M_{ab}L^2}{6}) \quad for: x < a$$

$$\delta = \frac{1}{EI}(\frac{M_{ab}(L - x)^3}{6L} - \frac{R_a x^3}{6} + \frac{Wa^2x^2(a + 3b)}{12b} - \frac{Wx^4(-a - b)}{24b} - \frac{Wax^3(a + 2b)}{12b} - \frac{Wx^5}{120b} + D_1 x + D_2) \quad for: a < x \leq a + b$$

$$\delta = \frac{1}{EI}(\frac{M_{ab}(L - x)^3}{6L} - \frac{R_b(L - x)^3}{6} + E_1 x + E_2) \quad for: x > a + b$$

please do not confuse E_1 & E_2 with the modulus of elasticity E

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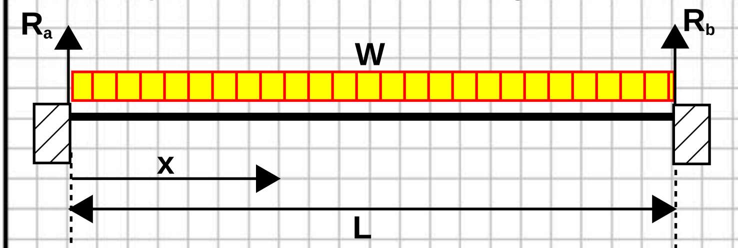
Encastre Beams

Uniformly Distributed Load Along Entire Beam



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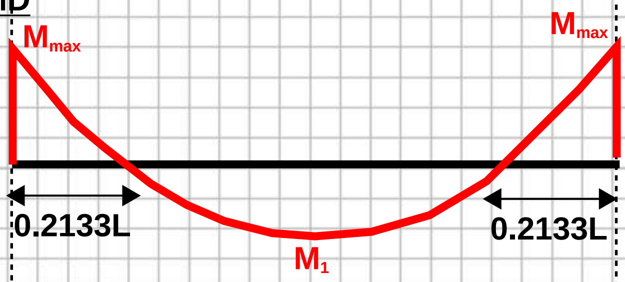
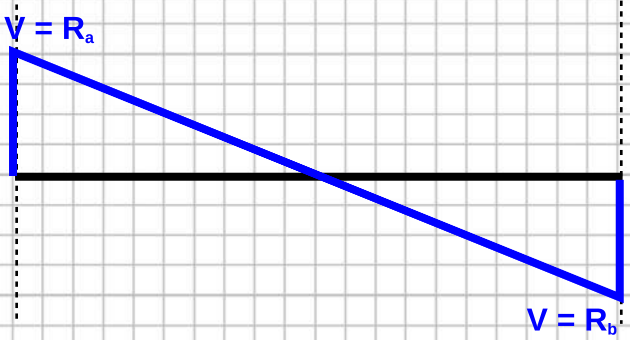


Shear



Inputs:

W : uniformly distributed load (kN/m)
L : Length of beam (m)
E : Modulus of Elasticity (N/mm^2)
I : Second Moment of Area (mm^4)
(for E & I you'll need to convert to kN and m)

Reactions:

$$R_a = R_b = \frac{WL}{2}$$

Moments:

$$M = \frac{W}{12}(6Lx - L^2 - 6x^2)$$

$$M_1 = \frac{WL^2}{24} \qquad M_{max} = \frac{WL^2}{12}$$

Shear:

$$V = W\left(\frac{L}{2} - x\right)$$

Deflection:

$$\delta = \frac{Wx^2}{24EI}(L-x)^2$$

$$\delta_{max} = \frac{WL^4}{384EI}$$

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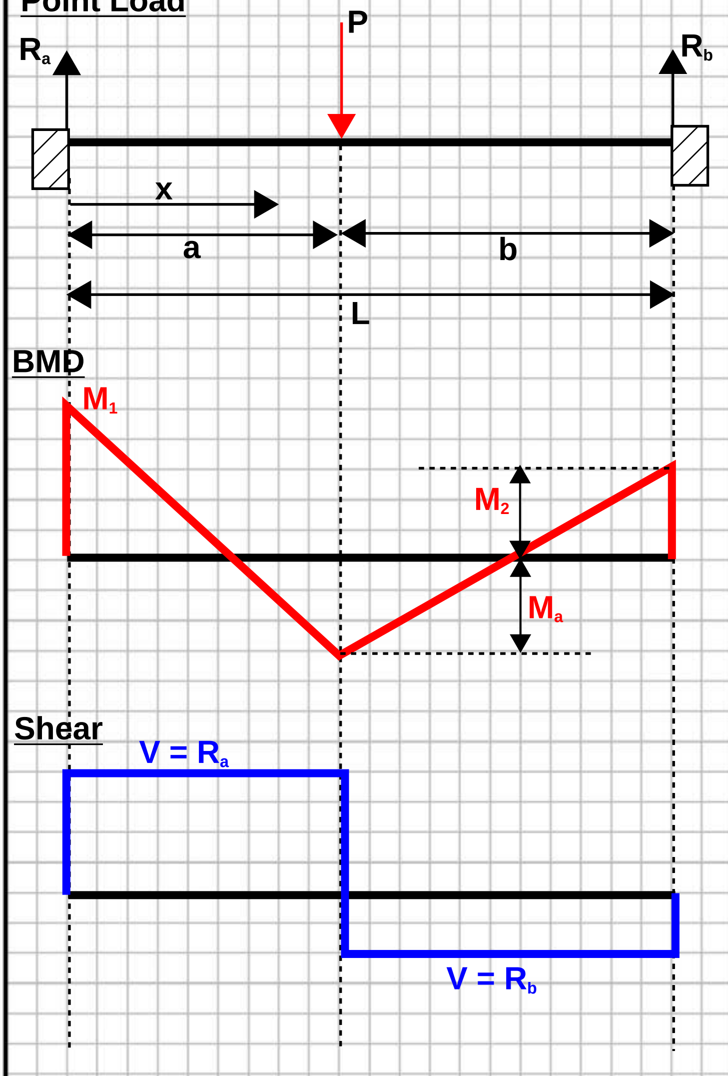
Encastre Beams

Point Load



Inputs:

P : Point Load (kN)
L : Length of beam (m)
a & b: Distances to the point load (m)
E : Modulus of Elasticity (N/mm²)
I : Second Moment of Area (mm⁴)
(for E & I you'll need to convert to kN and m)

Reactions:

$$R_a = \frac{Pb^2}{L^3}(3a+b)$$

$$R_b = \frac{Pa^2}{L^3}(a+3b)$$

Moments:

$$M_1 = \frac{Pab^2}{L^2} \qquad M_2 = \frac{Pa^2b}{L^2}$$

$$M_a = \frac{2Pa^2b^2}{L^3}$$

$$M = R_a x - \frac{Pab^2}{L^2} \quad for: x \leq a$$

$$M = -M_a + \frac{(M_2+M_a)(x-a)}{b} \quad for: x > a$$

Shear:

$$V = R_a \quad for: x \leq a$$

$$V = R_b \quad for: x > a$$

Deflection:

$$\delta = \frac{Pb^2x^2}{6EIL^3}(3aL - 3ax - bx) \quad for: x \leq a$$

$$\delta = \frac{Pa^2(L-x)^2}{6EIL^3}(3bL - 3b(L-x) - a(L-x)) \quad for: x > a$$

$$\delta_{max} = \frac{2Pa^3b^2}{3EI(3a+b)^2}$$

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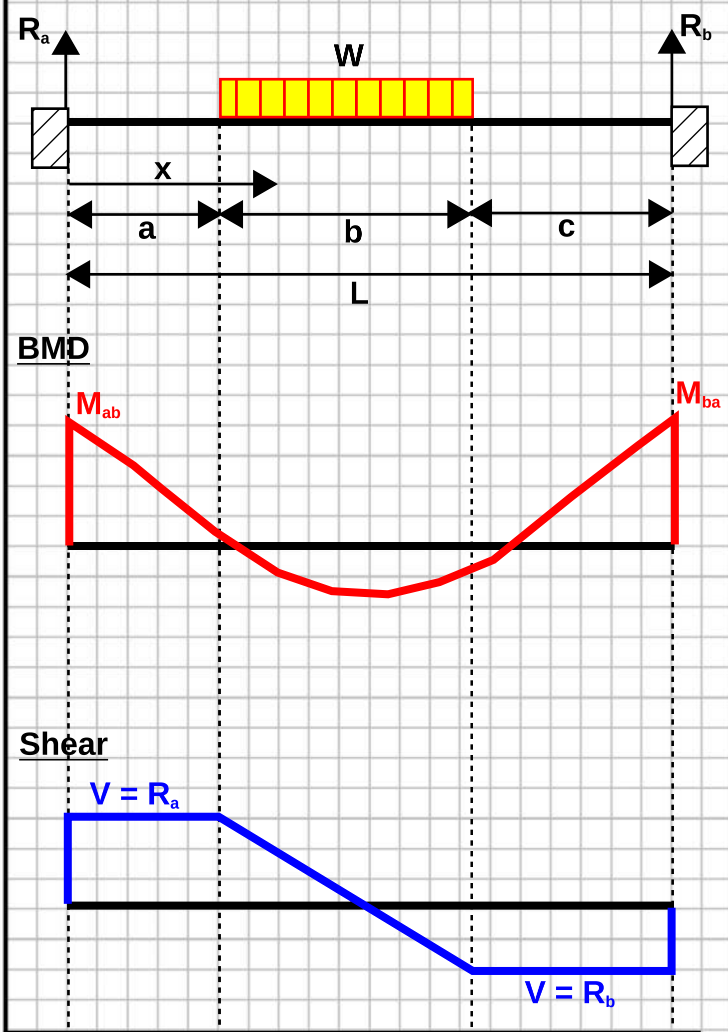
Encastre Beams

Partial UDL



Inputs:

W: Uniformly distributed load (kN/m)

L : Length of beam (m)

a,b & c: Distances and length of the UDL

E : Modulus of Elasticity (N/mm^2)

I : Second Moment of Area (mm^4)

(for E & I you'll need to convert to kN and m)

Intermediate Reactions:

$$R_a = \frac{Wb}{2L}(2c+b)$$

$$R_b = \frac{Wb}{2L}(2a+b)$$

final reactions found from the shear force diagram at x = 0 and x = L

Moments:

$$d = a + b$$
$$e = b + c$$

$$M_{ab} = \left|\frac{-Wb}{12L^2b}(e^3(4L-3e) - c^3(4L-3c))\right|$$

$$M_{ba} = \left|\frac{-Wb}{12L^2b}(d^3(4L-3d) - a^3(4L-3a))\right|$$

$$M' = \frac{M_{ba} - M_{ab}}{L}x + M_{ab}$$

$$M = M' - (R_a x) \quad for: x \le a$$

$$M = M' - (R_a x - \frac{W}{2}(x-a)^2) \quad for: a < x \le a+b$$

$$M = M' - R_b(L-x) \quad for: x > a+b$$

Shear:

$$V = R_a + \frac{M_{ab} - M_{ba}}{L} \quad for: x \le a$$

$$V_1 = R_a + \frac{M_{ab} - M_{ba}}{L} \qquad V_2 = R_b + \frac{M_{ba} - M_{ab}}{L}$$

$$V = V_1 - \frac{(V_1 + V_2)(x-a)}{b} \quad for: a < x \le a+b$$

$$V = -(R_b + \frac{M_{ba} - M_{ab}}{L}) \quad for: x > a+b$$

Deflection:

$$\delta = \frac{1}{EI}(\frac{M_{ba} - M_{ab}}{L} * \frac{x^3}{6} + \frac{M_{ab}x^2}{2} - \frac{R_a x^3}{6})$$

$$for: x \le a$$

$$\delta = \frac{1}{EI}(\frac{M_{ba} - M_{ab}}{L} * \frac{x^3}{6} + \frac{M_{ab}x^2}{2} - \frac{R_a x^3}{6} + \frac{W}{24}(x-a)^4)$$

$$for: a < x \le a+b$$

$$\delta = \frac{1}{EI}(\frac{M_{ba} - M_{ab}}{L} * \frac{x^3}{6} + \frac{M_{ab}x^2}{2} - \frac{R_b}{6}(L-x)^3 + E_1 x + E_2)$$

$$for: x > a+b$$

$$E_1 = -\frac{L}{2}(M_{ba} - M_{ab}) - M_{ab}L$$

$$E_2 = \frac{L^2}{3}(M_{ba} - M_{ab}) + \frac{M_{ab}L^2}{2}$$

please do not confuse E_1 & E_2 with the modulus of elasticity E

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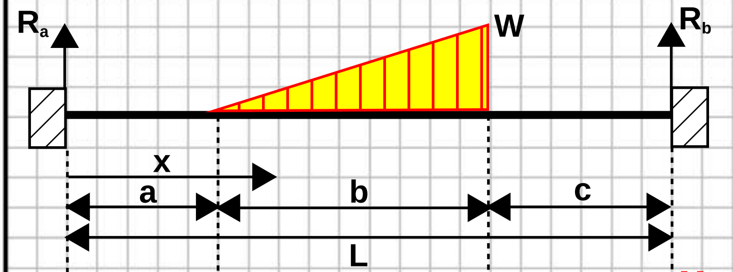
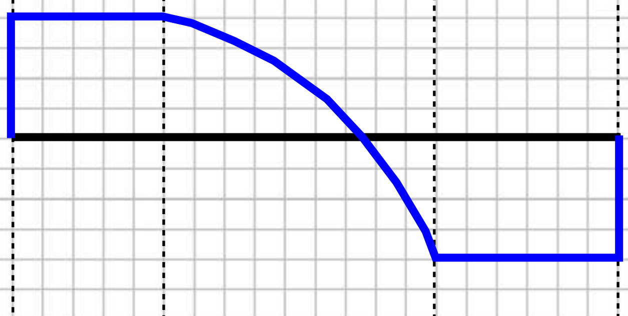
Simply Supported Beams

Triangular Load on Beam



BMD

Shear



Inputs:

W : uniformly distributed load (kN/m)
L : Length of beam (m)
E : Modulus of Elasticity (N/mm^2)
I : Second Moment of Area (mm^4)
(for E & I you'll need to convert to kN and m)

Intermediate Reactions:

$$R_b = \frac{Wb}{2L}(a + \frac{2}{3}b) \qquad R_a = \frac{Wb}{2} - R_b$$

final reactions found from the shear force diagram at x = 0 and x = L

Moments:

$$M_{ab1} = 30a^3 + 60a^2(b - L) + 5a(9b^2 - 16bL + 6L^2)$$

$$M_{ab2} = 2b(6b^2 - 15bL + 10L^2)$$

$$M_{ab3} = \frac{Wb}{60L^2}$$

$$M_{ab} = |(M_{ab1} + M_{ab2}) * M_{ab3}|$$

$$M_{ba1} = 30a^3 + 30a^2(2b - L) + 5ab(9b - 8L) + 3b^2(4b - 5L)$$

$$M_{ba2} = -\frac{-Wb}{60L^2}$$

$$M_{ba} = |M_{ba1} * M_{ba2}|$$

$$M' = \frac{M_{ba} - M_{ab}}{L}x + M_{ab}$$

$$M = M' - (R_a x) \quad for: x \leq a$$

$$M = M' - (R_a x) + \frac{W}{6b}(x - a)^3 \quad for: a < x \leq a + b$$

$$M = M' - R_b(L - x) \quad for: x > a + b$$

Shear:

$$V = R_a + \frac{M_{ab} - M_{ba}}{L} \quad for: x \leq a$$

$$V = R_a + \frac{M_{ab} - M_{ba}}{L} - \frac{W}{2b}(x - a)^2 \quad for: a < x \leq a + b$$

$$V = -(R_b + \frac{M_{ba} - M_{ab}}{L}) \quad for: x > a + b$$

Deflection:

$$\delta = \frac{1}{EI}(\frac{M_{ba} - M_{ab}}{L} * \frac{x^3}{6} + \frac{M_{ab}x^2}{2} - \frac{R_a x^3}{6})$$

$for: x \leq a$

$$\delta = \frac{1}{EI}(\frac{M_{ba} - M_{ab}}{L} * \frac{x^3}{6} + \frac{M_{ab}x^2}{2} - \frac{R_a x^3}{6} + \frac{W}{120b}(x - a)^5)$$

$for: a < x \leq a + b$

$$\delta = \frac{1}{EI}(\frac{M_{ba} - M_{ab}}{L} * \frac{x^3}{6} + \frac{M_{ab}x^2}{2} - \frac{R_b L x^2}{2} + \frac{R_b x^3}{6} + E_1 x + E_2)$$

$for: x > a + b$

$$E_1 = \frac{R_b L^2}{2} - L(\frac{M_{ba} - M_{ab}}{2} + M_{ab})$$

$$E_2 = \frac{R_b L^3}{3} - L^2(\frac{M_{ba} - M_{ab}}{6} + \frac{M_{ab}}{2}) - E_1 L$$

please do not confuse E_1 & E_2 with the modulus of elasticity E

If you want to model a triangular load going in the other direction (i.e. starts at W and drops to zero when going left-right). Simply set the following variables before running through the equations above
x = L - x
c = a
a = L - a - b

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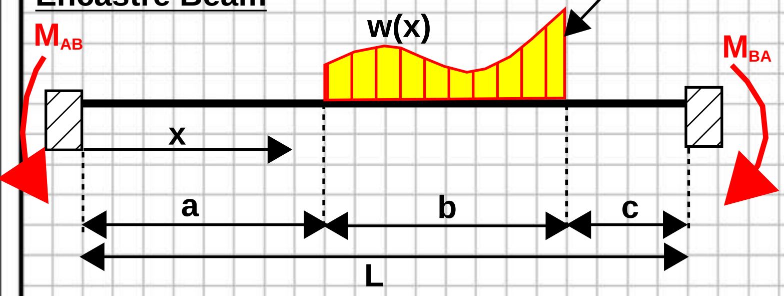
Calculate Fixed End Moments for Generic Cases

Encastre Beam



Generic load W expressed as a function of x i.e. w(x)

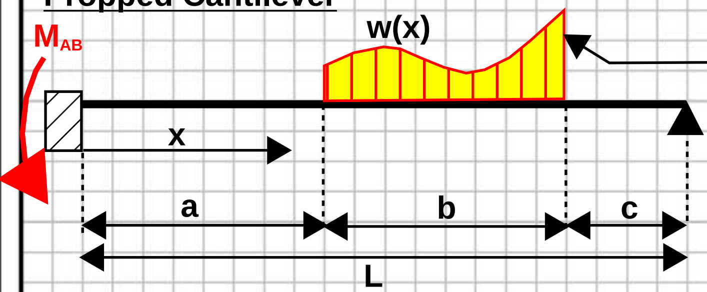
$$M_{AB} = \frac{1}{L^2}\int_a^{a+b} x(L-x)^2 * w(x)\ \delta x$$

$$M_{BA} = \frac{1}{L^2}\int_a^{a+b} x^2(L-x) * w(x)\ \delta x$$

We essentially use the formulas which gives us the fixed end moment for a point load on a encastre beam and we integrate this fixed end moment function over the length of the generic load w(x).

The fixed end moment for a point load on a beam at distance "a" from the left support is:
$M_{AB} = Pb^2a/L$
$M_{BA} = Pba^2/L$
These are the functions we integrate from
This is the function we integrate from "a" to "a+b"

Propped Cantilever



Generic load W expressed as a function of x i.e. w(x)

$$M_{AB} = \int_a^{a+b} w(x)(x - \frac{3x^2}{2L} + \frac{x^3}{2L^2})\ \delta x$$

We essentially use the formula which gives us the fixed end moment for a point load on a propped cantilever beam and we integrate this fixed end moment function over the length of the generic load w(x).

The fixed end moment for a point load on a propped cantilever beam at distance "a" from the left support is:
$M_{AB} = (P/L^2)(b^2a+[a^2b/2])$
This is the function we integrate from "a" to "a+b"